MSc project proposal
Topological Data Analysis of time-dependent networks

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Supervisors

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Description

Topological Data Analysis (TDA) [1]–[4] is a family of techniques gaining an increasing importance in the analysis and visualization of high-dimensional data in machine learning applications.

In this project, we will apply TDA techniques and persistent homology to time-dependent networks, in order to understand how the topological structure evolves over time in complex multilayer networks [5], [6].

There are two ways of obtaining time-dependent networks. Network data is available easily in many contexts: social networks and biological processes are two examples of systems evolving over time and that can be modelled as a graph. For instance, in social networks, links in ego networks have already been studied in the context of time-dependency [7].

The other large category is time series. It is possible to use a similarity measure to build a network from a set of time series taken from the same physical process. Although it could be applied to any set of time series, this has already been studied in the case of coupled oscillators (such as Kuramoto oscillators) [8], [9]. It is thus easy to find relevant datasets or to generate interesting data from physical simulations.

It is then possible to apply existing TDA and persistent homology techniques to the networks, taking into account the temporal dimension. Certain methods have already been implemented in topological data analysis libraries [10], [11], although they would have to be adapted to network data, and applied repeatedly to each time step. There is also a wide range of methods to explore, from the choice of the similarity measure, to the choice of filtration (in order to build a simplicial complex on the network), to the representation of topological structure. Each of these choices has a great influence on the final interpretation of the data, and may need to be adapted to each system.

PREREQUISITE COURSES/KNOWLEDGE

- SM7 Probability and Statistics for Network Analysis
- Topological Data Analysis and Persistent Homology[1]

COMPUTING REQUIRED?

Yes

DATA AVAILABLE?

Yes

References

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